

Model answers midterm Thursday 7 Oct 2016

- 1a) F : She buys fries
 B : she travels by bike
 C : she buys ice cream
 H : It is too hot

$$(F \wedge B) \rightarrow (C \vee H)$$

Also OK and logically equivalent:

$$(F \wedge B) \rightarrow (\neg H \rightarrow C)$$

$$((F \wedge B) \rightarrow C) \vee H$$

$$\neg H \rightarrow ((F \wedge B) \rightarrow C)$$

- 1b) G : Máxima visited Groningen
 W : Máxima visited Warffum
 R : Máxima visited farmerwâlde

$$\neg G \wedge (W \vee R)$$

2. Translation key:

d : Dawid

a : Anne

c : the book

b : Bhaskar

e : Elspeth

$H(x,y)$: x hated y

$T(x,y)$: x had treated y badly

$R(x,y)$: x read y

$B(x,y,z,w)$: x brought y from z to w

$P(x,y)$: x already possessed y

- a) $\neg H(d,e) \wedge \neg H(d,b) \wedge T(e,d) \wedge T(b,d)$. Also correct and logically equivalent:

$$\neg (H(d,e) \vee H(d,b)) \wedge T(e,d) \wedge T(b,d)$$

- b) $R(b,c) \rightarrow (B(d,c,b,a) \vee P(a,c))$

Also correct and logically equivalent:

$$R(b,c) \rightarrow (\neg P(a,c) \rightarrow B(d,c,b,a))$$

$$(R(b,c) \rightarrow B(d,c,b,a)) \vee P(a,c)$$

3a)	1. $A \leftrightarrow B$	
	2. $C \vee B$	
	3. C	
	4. $C \vee A$	\vee Intro: 3
	5. B	
	6. A	\leftrightarrow Elim: 1, 5
	7. $C \vee A$	\vee Intro: 6
	8. $C \vee A$	\vee Elim: 2, 3-4, 5-7

b)	1. $C \vee (B \wedge A)$	
	2. $\neg D \rightarrow \neg (B \vee C)$	
	3. $\neg D$	
	4. C	
	5. $B \vee C$	\vee Intro: 4
	6. $\neg (B \vee C)$	\rightarrow Elim: 2, 3
	7. \perp	\perp Intro: 5, 6
	8. $B \wedge A$	
	9. B	
	10. $B \vee C$	\vee Intro: 9
	11. $\neg (B \vee C)$	\rightarrow Elim: 2, 3
	12. \perp	\perp Intro: 10, 11
	13. \perp	\vee Elim: 9, 4-7, 8-12
	14. $\neg \neg D$	\neg Intro: 3-13
	15. D	\neg Elim: 14

(A somewhat shorter proof is possible by doing the \rightarrow Elim right after 3.)

3c) 1. $R(a,b) \wedge \neg R(b,c)$
 2. $c = b$
 3. $b = a$
 4. $\neg R(b,c)$ \wedge Elim: 1
 5. $R(a,b)$ \wedge Elim: 1
 6. $\neg R(b,b)$ $=$ Elim: 4, 2
 7. $\neg R(a,b)$ $=$ Elim: 6, 3
 8. \perp \perp Intro: 5, 7
 9. $\neg(b = a)$ \neg Intro

P	Q	R	$((Q \vee R) \wedge P) \rightarrow Q$			$(Q \vee (R \wedge P)) \rightarrow Q$		
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	F	T
T	F	T	T	T	F	T	T	F
T	F	F	F	F	T	F	F	T
F	T	T	T	F	T	T	F	T
F	T	F	T	F	T	T	F	T
F	F	T	T	F	T	F	F	T
F	F	F	F	F	T	F	F	T

Conclusion: $((Q \vee R) \wedge P) \rightarrow Q$ and $(Q \vee (R \wedge P)) \rightarrow Q$ are tautologically equivalent, because in all 8 situations they have exactly the same truth value

	Small(c)	Medium(c)	Large(d)	$(\text{Small}(c) \wedge \text{Medium}(c)) \rightarrow \neg \text{Large}(d)$	
Spurious	T	T	T	T	F
Spurious	T	T	F	T	T
	T	F	T	F	T
	T	F	F	F	T
	F	T	T	F	T
	F	T	F	F	T
	F	F	T	F	T
	F	F	F	F	T

Indeed the sentence $(\text{Small}(c) \wedge \text{Medium}(c)) \rightarrow \neg \text{Large}(d)$ is a logical possibility: it is true in at least one (even 6) non-spurious row

5 a) $(A \vee B) \rightarrow (C \rightarrow D)$ def. \rightarrow , 2x
 $\Leftrightarrow \neg(A \vee B) \vee (\neg C \vee D)$ \Leftrightarrow de Morgan
 $(\neg A \wedge \neg B) \vee \neg C \vee D$
 this sentence is in DNF

b. $\neg(A \wedge B) \leftrightarrow (B \wedge \neg C)$ def \leftrightarrow
 $[\neg\neg(A \wedge B) \vee (B \wedge \neg C)] \wedge [\neg(B \wedge \neg C) \vee \neg(A \wedge B)]$ \Leftrightarrow de Moyp
 $[(A \wedge B) \vee (B \wedge \neg C)] \wedge [\neg B \vee \neg\neg C \vee \neg A \vee \neg B]$ \Leftrightarrow $\neg\neg$ idemp.
 $[(A \wedge B) \vee (B \wedge \neg C)] \wedge [\neg B \vee C \vee \neg A]$ \Leftrightarrow distrib.
 $[(A \wedge B) \vee B] \wedge [(A \wedge B) \vee \neg C] \wedge [\neg B \vee C \vee \neg A]$ \Leftrightarrow distrib.
 $(A \vee B) \wedge (B \vee B) \wedge (A \vee \neg C) \wedge (B \vee \neg C) \wedge (\neg B \vee C \vee \neg A)$

This is already in CNF. It can be simplified further in various ways, e.g. to:
 $B \wedge (A \vee \neg C) \wedge (C \vee \neg A)$

- | | | | |
|-------|---|----|---|
| 6. a) | T | f) | T |
| b) | F | g) | T |
| c) | T | h) | T |
| d) | F | i) | F |
| e) | F | j) | F |

Abbreviation: $A \equiv (P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$ 5

Bonus

7

1.7 A

2 P

3 Q

4 $P \wedge Q$

5 A

6 \perp

7. $\neg Q$

8 $P \wedge \neg Q$

9 A

10 \perp

11 $\neg P$

12 Q

13 $\neg P \wedge Q$

14 A

15 \perp

16 $\neg Q$

17 $\neg P \wedge \neg Q$

18 A

19 \perp

\wedge Intro: 2, 3

\vee Intro: 4

\perp Intro: 5, 1

\neg Intro: 3-6

\wedge Intro: 2, 7

\vee Intro: 8

\perp Intro: 9, 1

\neg Intro: 2-10

\wedge Intro: 11, 12

\vee Intro: 13

\perp Intro: 14, 1

\neg Intro: 12-15

\wedge Intro: 11, 16

\vee Intro: 17

\perp Intro: 18, 1

20 $\neg \neg A$

\neg Intro: 1-19

21 $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$

$\equiv A$
 \neg Elim: 20